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STABILITY OF A WAKE BEHIND A FLAT PLATE IN A SUPERSONIC FLOW

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To a significant extent, the aerodynamics of an aircraft is determined by the nature of the flow in the wake behind it. Here, the resistance of the body in the flow may differ appreciably for the laminar and turbulent regimes [1]. Despite this, very few studies (especially experimental studies) of the stability of wakes have been conducted for supersonic velocities. This includes studies performed in the region of the former USSR.

Our experiments were conducted in the T-325 wind tunnel [2] at the Institute of Theoretical and Applied Mechanics (of the Siberian branch of the Russian Academy of Sciences). The Mach number of the incoming flow $M_\infty = 4$, the Reynolds number $Re_{1\infty} = 9\cdot 10^6$ m^{-1}. Flow stagnation temperature was about 290 K.

Stability characteristics in the transition were measured using a TPT-4 dc hot-wire anemometer with a sensor consisting of tungsten wire 6 μm in diameter and 1.2 mm long. Also used was a U2-8 selective amplifier, a V7-27A/1 voltmeter, and a spectrum analyzer (model 2010) made by the firm "Brüel and Kjaer."

The working part of the model was a thermally insulated symmetric steel plate 88 mm long, 10 mm thick, and 200 mm wide. One end of the plate was wedge-shaped, with an angle of 14°. A blunting of 0.1 mm was made on the plate's leading edge. The rear of the plate was also blunt and was tapered to a right angle. The plate was rigidly secured to the side walls of the working part of the tunnel and positioned at a zero angle of attack.

We obtained profiles of the mean voltage and standard deviation of voltage $\langle e \rangle$ on the sensor wire in the free viscous layer (the free stream of the boundary layer). The values were obtained at the longitudinal coordinate x = 12.5 and 15 mm, reckoned from the rear of the plate, and at x = 40, 60, and 80 mm in the wake proper.

Figure 1 shows dimensionless profiles of $\langle \bar{e} \rangle (y)$ (where y is the transverse coordinate, reckoned from the plane of symmetry of the wake; for greater clarity, the relations $\langle e \rangle (y)$ at x = 12.5 and 15 mm were converted to dimensionless form relative to the value of $\langle e \rangle_\infty$ at x = 40 mm). It is apparent that the difference between the maximum (in the cross section of the wake) pulsations in the wake and the pulsations on the symmetry axis of the wake decrease with an increase in the longitudinal coordinate.

At the same time, comparison of the relations E(y) and $\langle e \rangle y$ that we obtained showed that for each value of x the transverse coordinate y at which $\langle e \rangle$ is maximal across the wake corresponds roughly to the point of inflection in the relation E(y). This finding is consistent with the theory of instability of oblique airfoils (unfortunately, the shear layer is unstable in the calculations even in the inviscid approximation).

The position of the transition in the wake is determined fairly clearly from the position of the maximum of the voltage pulsations in the relation $\langle e \rangle (x)$ (see [3], for example). This is analogous to finding the position of the transition in a boundary layer by means of a hot-wire anemometer. Figure 2 also shows relations obtained in our experiments on the symmetry axis of the wake (y = 0) and in the layer with the maximum values of $\langle e \rangle$ along y (this layer is close to critical). The position of the transition determined in this manner corresponds to $x_t \approx 56$-59 mm and $Re_t = (u_e - u_0)x_t/\nu_0 \approx 0.3\cdot 10^5$ (the subscripts 0 and e pertain to the plane of symmetry of the wake and its boundary). In the near-critical layer, the transition occurs somewhat earlier (at $x_t \approx 56$ mm) than on the symmetry axis of the wake (where $x_t \approx 59$ mm) (as in [4]).

After the flow becomes turbulent (i.e., at x > 56 mm), the profile of perturbations in the wake (see Fig. 1) flattens out more rapidly than before the transition due to a reduction in the maximum perturbations in the layer. There is a more rapid

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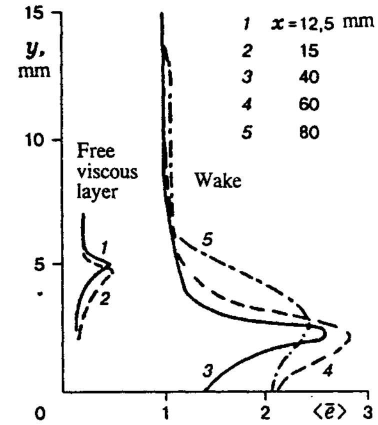

Fig. 1

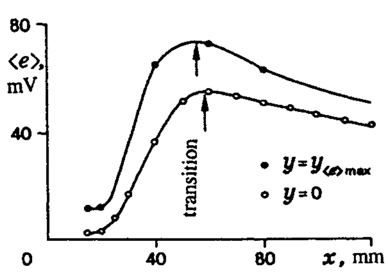

Fig. 2

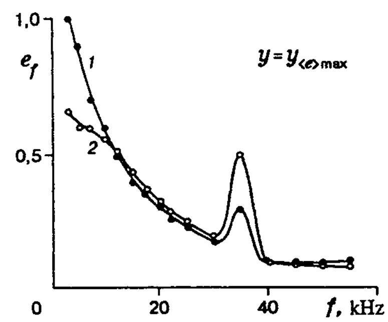

Fig. 3

increase in the thickness of the most disturbed part of the wake and the transverse coordinate with the maximum perturbations, which confirms the proposition advanced in [4, 5] that appreciable expansion of the wake is a sign of the transition.

The free viscous layer (the free stream of the boundary layer) changes in appearance from the boundary layer proper (immediately after the plate) to a more or less developed mixing layer (this finding agrees with [6]). Thus, it becomes quite interesting to compare the stability characteristics of the free viscous layer with the analogous characteristics of the boundary layer and then with the stability characteristics of the wake.

Figure 3 shows spectra of the energy of the pulsations on the sensor wire (the distributions of the amplitude of the perturbations e_f for different frequencies) in the free viscous layer for x = 12.5 and 15 mm – lines 1 and 2 (here and below, the amplitudes of the perturbations for different frequencies were made dimensionless relative to the amplitude of perturbations with a frequency of 3 kHz at x = 12.5 mm).

Figure 4 shows the growth of the perturbations $-\alpha_i = de/e\ \delta/dx$ in the free viscous layer (curve 1) in relation to their frequency f for x = 13.8 mm. Also shown (for comparison) is the increase in the perturbations in the boundary layer and at the end of the plate. These results were obtained previously with roughly the same regime of flow about a flat plate (curve 2). Here, δ is the thickness of the free viscous layer and the boundary layer. The relation showing perturbation growth has a second zero value (on the upper branch of the curve of neutral stability) at f = 140-150 kHz. It can be seen from Fig. 4 that the range of "unstable" frequencies decreases several-fold (at the expense of the high frequencies), although maximum perturbation growth in the free viscous layer is greater than in the boundary layer on the end of the plate. The maximum perturbations in the boundary layer and the end of the plate, ready to produce a transition in the boundary layer as a whole (as determined earlier, in the boundary layer on a flat plate at $M_\infty = 4$ and $Re_1 = 9 \cdot 10^6\ m^{-1}$, the beginning of the transition measured with the pitot tube (total-pressure tube) occurs at x ≈ 80-90 mm – which corresponds to the end of the plate-model being examined), are stabilized in the free viscous layer (these are perturbation with f ≥ 60 kHz).

Low-frequency perturbations, strongest in the free viscous layer, enter the latter from the boundary layer of the plate as extremely small disturbances and thus do not develop to the "transitional" level up to the mouth of the wake. In this sense

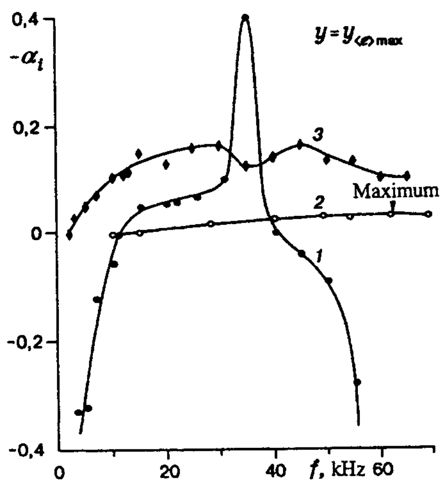

Fig. 4

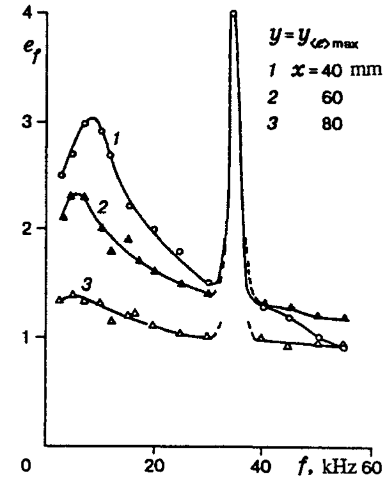

Fig. 5

(from the point of view of development of the transition in the boundary layer – free viscous layer – wake system), the free viscous layer is stable. The data obtained here supports the proposition in [7] that at M > 3 flow is laminar in the viscous layer (up to the mouth of the wake).

The development of perturbations in the wake proper is also shown in Fig. 4 (curve 3). The figure shows the degrees of growth $-\alpha_i$ of perturbations in the wake at x = 42.5 mm. These values are compared with the previously examined levels of growth in the free viscous layer and the boundary layer on the end of the plate. Whereas the range of "unstable" frequencies contracts sharply with the transition from the boundary layer to the free viscous layer, it expands significantly going from the viscous layer to the wake (also as a result of high frequencies).

Figures 5 and 6 show the pulsation energy spectra for the wake measured at x = 40, 50, 60, and 80 mm in the near-critical layer and on the line of symmetry of the wake. The characteristic maximum is seen in the spectral distribution of the pulsations both in the free viscous layer (see Fig. 3) and in the wake in the near-critical layer (Fig. 5) – the perturbations begin to increase sharply at $f \approx 35$ kHz. The Strouhal number, calculated from the frequency of this maximum, the thickness of the wake in the mouth region (determined from the velocity profile), and the velocity of the undisturbed flow, has the value $Sr = fb_0/u_\infty = 0.3$. A maximum in the pulsation spectrum was also seen in [8] in a study of the wake behind a flat plate at $M_\infty = 6$. The authors of [8] also obtained Sr = 0.3 and showed that this Strouhal number is universal for both subsonic and hypersonic flow velocities about a plate.

The data in Figs. 2, 5, and 6 give reason to believe that the linear development of perturbations in the wake (beginning with the mouth) at $Re_{1\infty} = 9 \cdot 10^6\ m^{-1}$ proceeds to $x \approx 40$ mm. The development then becomes nonlinear, and the transition begins at $x \approx 56$-59 mm. The energy spectra also change accordingly. While the fundamental tone, with $f \approx 35$ kHz, is dominant in the energy spectrum for a near-critical layer in the linear and nonlinear regions (similar results were obtained in [8]), the situation is found to be more complicated in the layer y = 0 (Fig. 6).

Perturbations with $f \approx 35$ kHz develop more intensively in the linear region than other perturbations (although their growth is considerably slower than in the critical layer), but perturbations with $f \approx 45$ kHz begin to grow rapidly in the nonlinear region. Most likely, the two maxima (at $f_1 \approx 5$-10 kHz and $f_2 \approx 35$ kHz) in the pulsation spectrum at $x \approx 40$-45 mm in the critical layer and in the layer y = 0 resonate between one another and a wave triplet is formed (in particular, a third wave with $f_3 \approx 45$ kHz). It can be seen from Fig. 6 that the growth of perturbations with frequencies corresponding to the two (at x = 40 mm) maxima is slowed considerably with the transition from the section x = 40 mm to the section x = 50 mm, while perturbations with $f_3 \approx 45$ kHz increase very rapidly, i.e., the energy jump in the wave triplet with the frequencies f_1, f_2, and f_3 goes from the first two waves to the third wave.

It is interesting that there was only one maximum in [8] in the near-critical layer (this maximum corresponding to the maximum with $f \approx 35$ kHz in our study), while in the layer y = 0 only the harmonic with the frequency 2f underwent significant growth (this difference from our findings is probably due to the fact that the thickness of the plate investigated in [8] was only 0.38 mm and the plate had a sharp back edge, i.e., the recirculation zone was almost nonexistent). It is possible

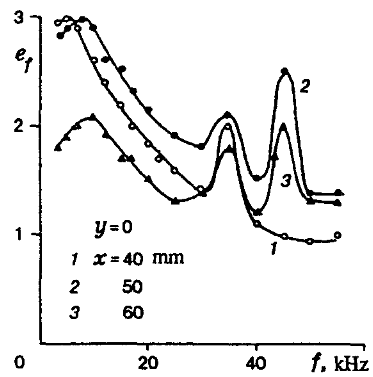

Fig. 6

that perturbations with the frequency 2f ≈ 70 kHz might also have increased in the layer y = 0 in the given experiments, but they were not recorded because the measurement range was limited to 3-60 kHz.

It is clear from Figs. 5 and 6 that while low-frequency perturbations predominate in the linear region of the wake (without consideration of the characteristic maximum), high-frequency perturbations grow more rapidly in the nonlinear region. As the transition approaches, the low-frequency perturbations decrease more rapidly than the high-frequency disturbances and the spectrum flattens out. The associated energy redistribution causes all of the frequencies of the energy pulsations to become equalized.

Thus, for $M_\infty = 4$, we studied the development of perturbations in the free viscous layer and the wake proper behind a flat plate with a symmetric wedge-shaped front having a sharp leading edge and a blunt (tapered to a right angle) rear. Characteristics of flow stability were obtained in the free viscous layer and the wake. A characteristic maximum was found in the spectral distribution of the pulsations corresponding to the Strouhal number 0.3 (calculated from the frequency of the same maximum). A wave triplet satisfying a resonance frequency relation was observed in the plane of symmetry of the wake in the nonlinear region of perturbation growth.

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